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DISCRETE - CONTINUOUS OPTIMIZATION FOR MECHATRONIC SYSTEM DESIGN

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ABSTRACT

The positioning of the actuators, together with the determination of the controller gains in the design of actively controlled structures are very important in the characterization of mechatronic systems. In this paper is formulated, described and solved a multi-objective discrete-continuous optimization problem that determines the optimal position of piezoelectric actuators/sensors along a flexible structure, as well as the controller gains.

For that purpose, a discrete-continuous optimization problem is defined, and optimization methods based on Genetic Algorithms (GAs) are used for the determination of the discrete positions of the actuators/sensors along the structure. The goal of this optimization step is to minimize the control energy applied to the system. Simultaneously, a second optimization step is proceeded, in order to determine the gain values of the controller, by using classical optimization techniques (SUMT), aiming at maximizing the structural damping without surpassing the maximum electric load admitted by the actuators.

NOMENCLATURE

C - kinetic energy
U - potential energy
W_e - work done by electrical forces
W_m - work done by magnetic forces
δW - Virtual work done by external forces
P_b - Body forces
P_s - Surface forces
P_c - Concentrated load
Q - Surface electric charge
u - Displacement
ϕ - Electric potential
N_x - Mechanical interpolations functions
N_ϕ - Electrical interpolation functions
ρ - Density
V - Volume
S - Surface
$[]_s$ - Subtitles s refer to the structure
$[]_p$ - Subtitles p refer to the piezoelectric actuators
$[]^T$ - Transpose vector or matrix
T - Stress
S - Strain
D - Electric displacement
E - Electric field
$[c^E]$ - Elasticity matrix (constant electric field)
$[e]$ - Dielectric permittivity matrix
$[\varepsilon^S]$ - Dielectric matrix (constant mechanical strain)
L - Finite element length
t - Thickness
w - width
m - Elemental mass matrix
D - Proportional damping matrix
$[k_s]$ - Structural stiffness matrix
$[k_{uu}]$ - Piezoelectric stiffness matrix,
$[k_{u\phi}]$ $[k_{\phi u}]$ - Electromechanical matrix
$[k_{\phi\phi}]$ - Piezoelectric capacitance matrix
$\{u_f\}$, F - External forces
$[G_P]$ - Proportional gain matrix
$[G_D]$ - Derivative gain matrix
$[T_s]$ - Sensor-actuator distribution matrix

INTRODUCTION

The study of smart structures has received great attention along the last decade in regard to their ability of improving the performance of conventional flexible structures, especially for aerospace and aeronautic applications.

Different methodologies have been used for modeling smart structures, however one of the most useful is the Finite Element Method (FEM), first presented for piezo mechanical systems by Allik and Hughes[1]. Later this formulation was extend for different structural elements[2][3][4].

The FEM models for mechatronic systems can be represented by state space equations as presented by Hagood et al. [5]. Different control systems have been proposed for smart structures [6][7], however the position, the number and dimension of the actuators in the structure are frequently defined by personal experience, which can imply serious difficulties when complex structures are concerned. Some considerations about position and size of actuators are presented in several research works [8][9][10][11]

The topic of the present contribution has been studied by various authors: Gabbert et al. [10] used classical optimization for determining controller parameters; Kirby III and Matic [11] worked with genetic algorithms to determine optimal actuator size and location for two piezoceramic actuators bonded to a cantilever beam; Lammering et al. [12] used the electric potentials to minimize the control effort in the optimal placement of piezoelectric actuators in adaptive truss structures.

Some of the basic ideas used in this paper were first described in a previous work by Lopes et al. [13]. The design procedure is based on a multi-objective discrete-continuous optimization problem that determines the optimal position of piezoelectric actuators/sensors along a flexible structure, as well as the controller gains.

In the determination of optimal discrete positions of actuators/sensors, Genetic Algorithms (GA) techniques were used. The goal was to minimize the control effort applied to the system with respect to a pre-determined vibration reduction requirement. As constraint functions, the maximum number of actuators and the actuator position surface are taken into account. The elements of a discrete position matrix were considered as design variables.

Classical techniques of optimization (SUMT) are used simultaneously together with the discrete optimization scheme for the determination of the continuous gain values of the controller (Zhu et al. [14]). The objective here is to maximize damping introduced in the structure by means of piezo –actuators. Restrictions concerning the maximum electric load admitted by the actuators/sensors must be satisfied in the optimization procedure.

A proportional derivative (PD) control system was used in the design of the controller, which was applied to a flexible cantilever beam.

FINITE ELEMENT MODEL

The finite element technique is formulated using the *Virtual Work* principle [1] that is based in the virtual continuous displacement under external electrical and mechanical forces, as represented by equation (1):

$$\int_{t_1}^{t_2} \left[\delta\left(C - U + W_e - W_m\right) + \delta W \right] dt = 0 \qquad (1)$$

where: t_1 and t_2 are two different time instants

Rewriting equation (1) by replacing the energy terms by integrals of external forces, and assuming that the magnetic work is not considered for piezoceramics, we have:

$$C = \int_{V_s} \rho_s \ddot{u} dV + \int_{V_p} \rho_p \ddot{u} dV \qquad (2)$$

$$U = \int_{V_s} S^T T dV + \int_{V_p} S^T T dV \qquad (3)$$

$$W_e = \int_{V} \delta E D\, dV + \int_{S} \delta\phi\, Q\, dS \qquad (4)$$

$$\delta W = \int_{V_s} \delta u^T P_b dV + \int_{S_s} \delta u^T P_s dS + \delta u^T P_c + \int_{S_p} \delta\phi Q dS \qquad (5)$$

This procedure gives the following equation for a coupled electromechanical system:

$$\int_{t_1}^{t_2} \left[\begin{array}{l} \delta \left(\begin{array}{l} \int_{V_s} \rho_s \ddot{u} dV + \int_{V_p} \rho_p \ddot{u} dV - \int_{V_s} S^T T dV - \\ \int_{V_p} S^T T dV + \int_{V_p} E^T D dV \end{array} \right) + \\ \int_{V_s} \delta u^T P_b dV + \int_{S_s} \delta u^T P_s dS + \delta u^T P_c + \int_{S_p} \delta\phi Q dS \end{array} \right] dt = 0 \qquad (6)$$

where: P_b represents the body forces, P_s represents the surface forces, P_c represents the concentrated load, Q is the surface electrical charge in the actuators, ρ is the density of the element, and the subtitles s and p refer to the structure and piezoelectric actuators, respectively.

From equation (6) it is possible to write the variational equation for an Euler - Bernoulli piezo-actuated beam. The reference system presented in Figure 1 is used, and the following assumptions are taken into account:

1. The electric potential is constant in the piezoelectric surface, and has a linear variation along its thickness.

2. The adhesive thickness is not considered.
3. A perfect mechanical coupling between the actuator and the structure is obtained.

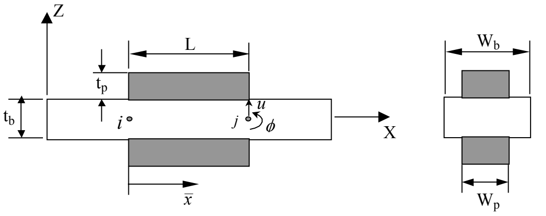

Figure 1: Reference System.

Interpolation functions are written for the displacement $\{u\}$ and electrical potential $\{\phi\}$, which are expressed in terms of the nodal value, *i*. This procedure leads to the formulation of the electroelastic matrix.

It is considered that the interpolation functions have the required property for the correct numerical convergence of the problem. It is possible to relate the mechanical stress with the nodal displacement by a derivation operator (L_u), and the electric field with the electric potential by a gradient operator, as shown, respectively, by equations (7) and (8):

$$\begin{aligned} S(x,t) &= [L_u][N_u(x)]\{u_i(t)\} \\ S(x,t) &= [B_u(x)]\{u_i(t)\} \end{aligned} \tag{7}$$

$$\begin{aligned} E(x,t) &= -\nabla[N_\phi(x)]\{\phi_i(t)\} \\ E(x,t) &= -[B_\phi(x)]\{\phi_i(t)\} \end{aligned} \tag{8}$$

It is now necessary to use the piezoelectric constitutive relations given by equations (9) and (10).

$$\{T\} = [c^E]^T\{S\} + [e]\{E\} \tag{9}$$

$$\{D\} = [e]^T\{S\} - [\varepsilon^S]\{E\} \tag{10}$$

Substituting equations (7) and (8) in equations (9) and (10), we have:

$$\{T\} = [c^E]^T[B_u]\{u_i\} + [e](-[B_\phi]\{\phi_i\}) \tag{11}$$

$$\{D\} = [e]^T[B_u]\{u_i\} - [\varepsilon^S](-[B_\phi]\{\phi_i\}) \tag{12}$$

Allowing arbitrary variations of $\{u\}$, two equilibrium matrix equations are obtained in generalized coordinates, as shown by equations (13).

$$\begin{aligned} &([m_s]+[m_p])\{\ddot{u}_i\} + ([k_{uu}]+[k_s])\{u_i\} - [k_{u\phi}]\{\phi_i\} = \{F\} \\ &[k_{\phi u}]\{u_i\} - [k_{\phi\phi}]\{\phi_i\} = \{Q\} \end{aligned} \tag{13}$$

where the matrices above are defined for the structure and piezoelectric path elements accordingly (for more details see Pereira and Steffen [15]).

Finally, it is possible to write a global equation for the coupled electromechanical system by making a simple nodal summation of the local contribution of each element, as given by equation (14):

$$\begin{bmatrix} [M] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{u\phi}] \\ [K_{\phi u}] & [K_{\phi\phi}] \end{bmatrix} \begin{Bmatrix} u \\ \phi \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{Q\} \end{Bmatrix} \tag{14}$$

where:

$[M]$ – Mass matrix including the structure and piezoelectric patches contributions,
$[K_{uu}]$ – stiffness matrix including the structure and piezoelectric patches contributions,
$[K_{u\phi}]$ – global electromechanical matrix,
$[K_{\phi\phi}]$ – global piezoelectric capacitance matrix.

CONTROL STRATEGY

Before considering the control scheme it is necessary to define a subset of coordinates corresponding to the placement of the actuators/sensors. This subset is related to the actuators/sensors positioning $[\{u_s\},\{\dot{u}_s\}]$ to the general coordinates $[\{u\},\{\dot{u}\}]$ by a binary distribution matrix $[T_s]$ as shown in equation (15).

$$\{u_s\} = [T_S]\{u\} \text{ and } \{\dot{u}_s\} = [T_S]\{\dot{u}\} \tag{15}$$

$[T_s]$ is a *2m* x *n* matrix, where *m* is the number of actuators/sensors in the structure, and *n* is the total number of degrees of freedom (DOF) considered for the system. A "zero" input means that no actuator/sensor is placed at the corresponding DOF, and a "one" input means that an actuator/sensor is placed at that particular DOF.

Proportional Derivative Controller

After defining $[\{u_s\},\{\dot{u}_s\}]$ and by considering a usual proportional derivative (PD) controller, it is possible to write a control law based on the electrical potential ϕ, which can be calculated as a function of measurable outputs, as shown by equation (16).

$$\{\phi\} = [G_P]\{u_s\} + [G_D]\{\dot{u}_s\} \tag{16}$$

where $[G_P]$ and $[G_D]$ are the proportional and derivative controller gain matrices, respectively.

Proportional damping was introduced in the system in order to improve the system controllability. The system equation of motion, taking into account the control, can be obtained by introducing equations (15) and (16) into the first part of equation (14):

$$[M]\{\ddot{u}\} + [D^*]\{\dot{u}\} + [K^*]\{u\} = \{F\} \tag{17}$$

where:

$$[D^*] = [D] + [K_{u\phi}][G_D][T_S] \tag{18}$$

$$[K^*] = [K_{uu}] + [K_{u\phi}][G_P][T_S] \tag{19}$$

where $[D]$ is a proportional damping matrix.

Equation (17) is then integrated with respect to time by using the Newmark scheme [16].

As $\{\phi\}$ can be calculated through the control law equation, the system is considered as a voltage controlled one, and the second part of equation (14) can be used to calculate the actuator surface charge $\{Q\}$:

$$\{Q\} = ([G_D][T_s])\{\dot{u}_s\} + ([K_{\phi u}] + [G_P][T_s])\{u_s\} \tag{20}$$

Equation (17) can be rewritten in the state space form, as represented by equation (21):

$$\{\dot{z}\} = [A]\{z\} + [B]\{u_f\} \tag{21}$$

where $\{z\} = [\{u\}\,\{\dot{u}\}]^T$ is the state vector, $[A]$ is the state matrix, $[B]$ is the input matrix, $\{u_f\}$ is the external force, and

$$[A] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K^*] & -[M]^{-1}[D^*] \end{bmatrix} \tag{22a}$$

$$[B] = \begin{bmatrix} [0] \\ [M]^{-1} \end{bmatrix} \quad ; \quad \{u_f\} = \{F\} \qquad \text{(22b; 22c)}$$

Natural frequencies and vibration mode shapes can be computed from the dynamic matrix $[A]$. From equation (21) it is possible to determine the system performance for a set of controller parameters.

Control Project

In this paper two different methodologies were used for the controller design.

First approach: it aims at maximizing the vibration reduction by repositioning the poles, which are expected to be as far left in the complex plane as possible. At the same time, the value of the driving voltage cannot surpass a given maximum value that depends on the PZT characteristics. For this purpose, the second part of equation (14) can be used as a constraint equation in the controller optimal design.

Second approach: it uses the linear quadratic regulator (LQR), for which the optimal solution can be obtained by the minimization of a performance criterion given by:

$$J = \int_{t_0}^{t_1} \left(u^T Q_1 u + \phi^T Q_2 \phi\right) dt \tag{23}$$

where Q_1 and Q_2 are weighting matrices, which are to be chosen for optimal control purposes. Gabbert et al.[10] considers that the mechanical energy is physically restricted. In the case of the present paper, constraint equations are written in such a way that vibration amplitude is limited to a given prescribed value. This way, the first term Q_1 of equation (30) can be neglected in the minimization process. The resulting cost function can be interpreted as an electrical work done by external forces to control the vibrations of the system. This situation corresponds to $Q_2 = K_{\phi\phi}$.

$$J = \int_{t_0}^{t_1} \left(\phi^T K_{\phi\phi} \phi\right) dt \tag{24}$$

Consequently, equation (24) is used in this paper for the two different approaches: in the first one a discrete objective function is to be minimized to determine the optimal actuator positions (the controller gains are obtained from the poles positioning); in the second approach, the equation is used to determine the position of the actuators together with the optimal gain matrices of the controller.

OPTIMIZATION PROBLEM

In the design of smart structures it is required the determination of the optimal position of the actuators/sensors elements and, simultaneously, the gain matrices of the controller. This can be achieved through the solution of a discrete-continuous optimization problem, where the non-linear optimization problem can be defined as follows:

$$\text{Minimize:} \quad F(x) \tag{25}$$

$$\text{subject to:} \quad g_i(x) \le 0 \quad i = 1,\dots,m \tag{26}$$

$$\text{variables:} \quad x_j \in [0 \text{ or } 1] \quad j = 1,\dots,n_D \tag{27}$$

$$x_{j,lb} \le x_j \le x_{j,ub} \quad j = n_D + 1,\dots,n \tag{28}$$

where: $n = n_D + n_C$, n_C is the number of continuous variables, n_D is the number of discrete variables, $x_{j,lb}$ and $x_{j,ub}$ are the bounds of the continuous design variables.

In this case two optimization goals related to the dynamical behavior of the system must be achieved simultaneously: the first is the minimization of the control effort, as represented by the electric work, and the second one is the maximization of the structural damping. As it was mentioned above, two different approaches are used to deal with this problem.

In the first approach, the two criteria to be minimized are represented mathematically by the following equations:

$$J = \min\left[\int_{t_0}^{t_1} \left(\phi^T K_{\phi\phi}\phi\right) dt\right] \tag{29}$$

and

$$f_{obj}(x) = \min_k \left[\operatorname{Re} \lambda_k([A])\right] \tag{30}$$

where: $\lambda_K([A])$ is the *kth* eigenvalue of $[A]$, *Re* represents the real part of this eigenvalue, J represents the discrete objective function, and f_{obj} is the continuous objective function.

In order to write the constraint equations, the surface charge in the actuator is written as:

$$[K_{\phi u}]\{u\} + [K_{\phi\phi}]\{\phi\} = \{Q\} \tag{31}$$

where $\{\phi\}$ is given by equation (16). However, the surface charge is limited by a given value that is a function of the material proprieties. It is possible to write a set of n_p inequality constraint equations by using equation (31):

$$\{Q\} \le \{Q_{\max}\} \tag{32}$$

This means that equation (32) is to be respected while the structural damping is maximized along the optimization procedure. In order to limit the mechanical displacement of the structure, a new constraint function is added, i.e, the vibration amplitude at a given DOF (reference DOF), when the control is active, must be lower than a prescribed value, as compared with the amplitude of the non-controlled system:

$$\frac{\{u_{ref}\}}{\{u_0\}} - C_R \le 0 \tag{33}$$

where: $\{u_{ref}\}$ is the displacement vector of the reference DOF for the controller system, and $\{u_0\}$ is the displacement vector for the reference DOF for the non-controlled system.

Simultaneous optimization of structural and controller parameters is conducted by iteratively executing the structural and control optimization processes [14], according to the following interdependent steps:

1. J is minimized with respect to the discrete variables (x_j, $j = 1,\dots,n_D$),
2. $f_{obj}(x)$ is minimized with respect to the continuous variables (x_j, $j = n_D + 1,\dots,n$).

These steps are repeated until convergence is achieved.

The second approach minimizes the same objective function (29) with respect to the continuous and discrete variables, The same constraints are taken in account.

The first step is performed as above and in the second step J is minimized with respect to the continuous variables, instead of $f_{obj}(x)$.

Both approaches are performed by using the scheme shown in Figure 2.

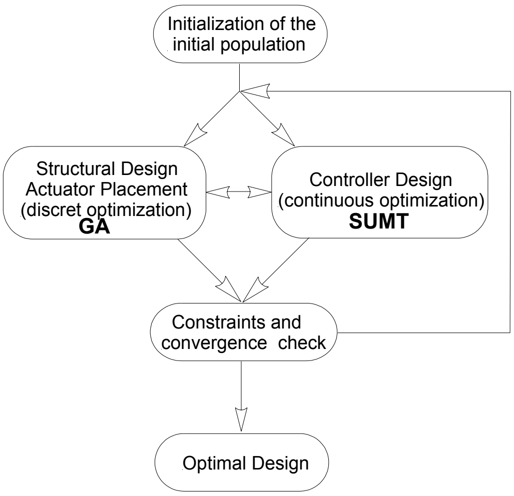

Figure 2: Optimization scheme.

The discrete optimization was performed by using Genetic Algorithms. This is a pseudo random search technique based on Darwin's evolution theory. The basic principle of the method is that an initial population evolves over generations to produce a new and, hopefully, better design. The elements (designs) of the initial population are randomly or heuristically generated. A basic genetic algorithm uses four main operators, namely: *evaluation, selection, crossover* and *mutation*. In this paper the software *GAOT – the Genetic algorithm Optimization Toolbox for MatLab 5* [17] was used.

The continuous optimization problem is solved by using classical sequential unconstrained minimization techniques (SUMT). For this purpose, the Augmented Lagrange Multiplier Method is used to define a pseudo–objective function, which takes into account the constraint functions. The unconstrained minimization is performed by using the *BFGS (Broyden-Fletcher-Goldfarb-Shanno)* method. The one dimensional search is performed by using polynomial interpolation and golden section techniques.

CASE STUDY

For illustrative purposes, the design of a simple structure was simulated.

System Properties

Consider a clamped-free Euler-Bernoulli beam with piezoelectric elements bonded on its surface. Mechanical and piezoelectric proprieties are presented in tables (1) and (2).

Table 1: Mechanical Properties

	Structure	**Actuator/Sensor**
Material	Aluminum	PZT – PSI-5A
Dimensions	$30x500x5$ mm	$30x25x0{,}5$ mm
Young's Modulus	70 Gpa	62 GPa
Density	2710 Kg/m^3	7750 Kg/m^3

Table 2: Piezoelectric Properties

Piezoelectric Strain Coefficient (m/V)	$d_{31} = -190.10^{-12}$
	$d_{33} = 390.10^{-12}$
Dielectric Constant (F/m)	$\varepsilon_{33}^{s} = 7{,}33.10^{-9}$
Relative Dielectric Constant	$K^T = 1800$

The system was discretized in 20 equal length elements. Each element has 02 nodes, with 02 DOF per node, namely: (u) vertical displacement and (ϕ) angular displacement. A zone, where the placement of PZTs is accepted for design purposes is defined between elements 1 to 10 , as shown in Figure 3.

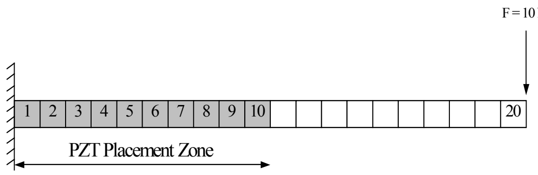

Figure 3: Clamped - free beam.

The system was excited by an impulsive force of 10 N during 0.1 seconds at its free end. A vibration amplitude reduction greater than 70% was imposed to the controlled system, as compared with non-controlled case.

$$\frac{\{u_{ref}\}}{\{u_0\}} - 0{,}3 < 0 \qquad (34)$$

Simulation Results

Table (3) presents the two different cases studied, according to the control strategy used.

Table 3: Cases studied

	Maximum Number of PZTs	**Placement Zone (Elements)**	**Control Method**
Case 1	05	1-10	Pole
Case 2	05	1-10	LQR

In all cases the optimization process converged successfully; all constraints were respected, and the results are presented below.

Case 1: the optimum solution uses 04 actuator positioning on elements 1 to 4. Figure 4 shows the displacement of the beam free end and the electric charge on the critical actuator.

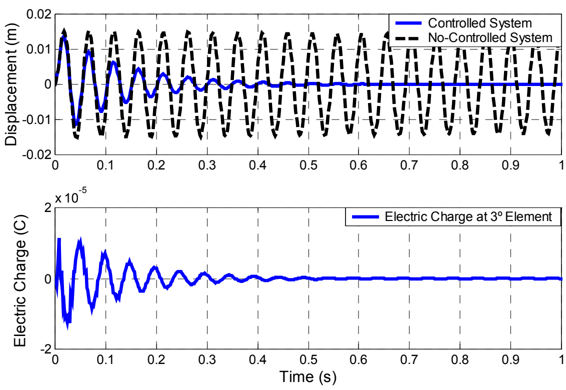

Figure 4: Displacement and Electric Charge for Case 1.

Figure 5 shows the evolution of the mean of the population and the best individual. It is possible to observe the convergence of the GA and its ability to manage with unexpected values parameter (10^{a} generation).

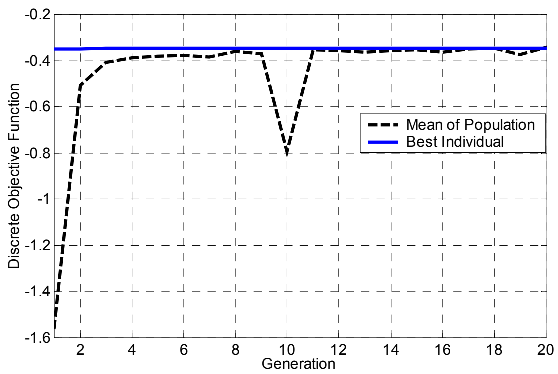

Figure 5: Evolution of discrete optimization process.

The continuous optimization process can be observed by the pole position evolution along the optimization process, as shown in Figure 6. The process is capable of working with bad initial control designs and it is able to focus its priority on the critical poles.

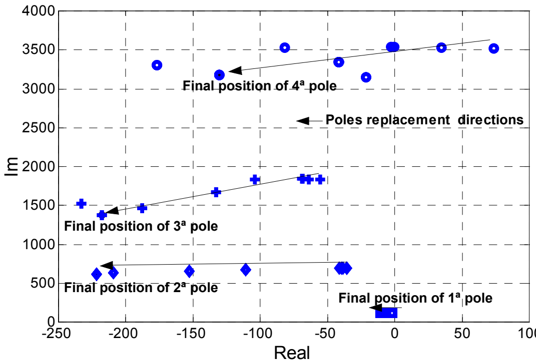

Figure 6: Pole position evolution on the continuous optimization process.

Case 2: the optimization processes impose the use of 5 actuators, positioning them on the elements 1 to 5. The control system is effective in the whole frequency band as show in Figure 7, specially on first mode.

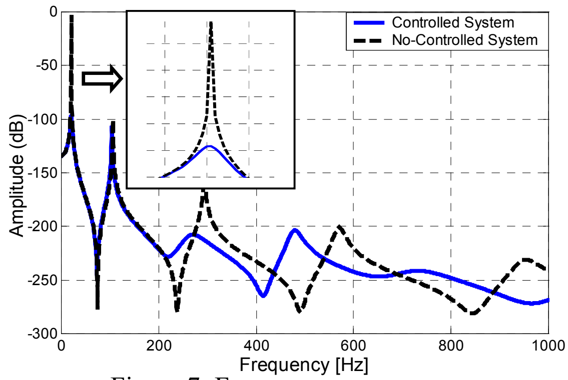

Figure 7: Frequency response.

This approach does not use pole replacement, but, through an indirect way, it is possible to observe the continuous optimization process by accompanying the movement of the poles in the complex plane (Figure 8).

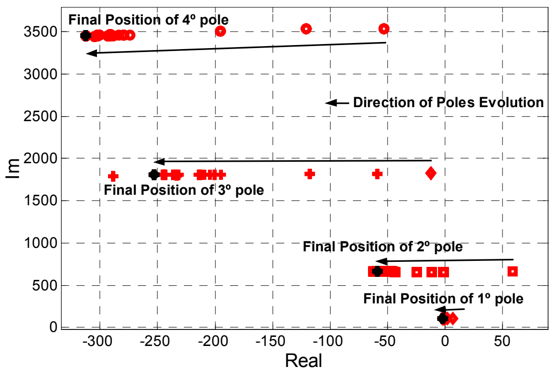

Figure 8: Pole position evolution on the continuous optimization process.

CONCLUSIONS

A discrete-continuous optimization problem was defined aiming at the determination of optimal piezoelectric actuators/sensors positions together with the determination of optimal controller gains for an Euler-Bernoulli beam. Genetic Algorithms were used to handle discrete design variables and a classical sequential unconstrained minimization technique was used to deal with continuos design variables. The hybrid optimization scheme showed to be very effective in solving the problem. When the number of candidate positions for the PZTs is large compared to the number of available PZTs, computing time increases significantly. The authors consider that the methodology presented has good potential for mechatronic structures design.

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